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Quantify Uncertainty in Scanline Estimates of Volumetric Fraction of Anisotropic Birmocks

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ABSTRACT: An analytical solution on the uncertainty in the estimates of volumetric fraction of different compositions in a Bimrock is presented in this paper. The derivation was developed based upon the concept of representative volumes. To tackle the anisotropic orientation of blocks, this study developed the concept of a parallelogram representative volume element that contains an ellipse. Our results demonstrate that the uncertainties in the estimates depend upon the aspect ratio, orientation, and diameter of blocks and the level of volumetric fraction. We further simplified the analytical expression in terms of intercept length. Our results were verified through numerical simulation albeit preliminary.

1. INTRODUCTION

We have previously developed an analytical solution to the uncertainty of volumetric fraction estimates of isotropic, Block-in-Matrix, (Bimrocks) [1]. Bimrocks are defined as “a mixture of rocks, composed of geotechnically significant blocks within a bonded matrix of finer texture” [2]. Bimrocks thus encompass a wide range of geologic materials including, for example, melanges, fault rocks, landslide debris, and glacial till. Their overall mechanical behaviors are highly dependent on their volumetric block fraction [2][3][4].

Three categories of measurement methods have been used in estimating the volumetric fraction, V_f , of Bimrocks, namely, one-dimensional (linear measurement and borehole), two-dimensional (image analyses and window mapping) and three-dimensional (sieve analyses). Although sieve analysis is the most accurate method for laboratory-scale studies, separation of blocks from the weaker matrix is not always possible, affecting by factors such as the number and size of blocks, and the degree of contact strength between blocks and matrix [1].

According to the basic principles of stereology, if the sampling is under IUR - isotropic, uniform and random conditions- such that all portions of the structure are equally represented (uniform), there is no conscious or consistent placement of measurement regions with respect to the structure itself to select what is to be measured (random), and all directions of measurement are equally represented (isotropic) [5], namely, the results be the same regardless of the dimension of a measurement method. Or, simply put

$$V_f = A_f = L_f = P_f \qquad (1)$$

where V_f is volumetric fraction, A_f is area fraction, L_f is linear fraction, and P_f is point-count fraction.

Thus, in this paper we interchange the use of V_f, A_f and L_f.

Scanline is one of the most efficient and economical method for estimating V_f, and may be the only way such as in the case of sampling through drilling. Scanlines estimate V_f by dividing the total cumulative intercept length, or block length, with the total scanlin length. Thus its operation and processing is rather straightforward. But scanline use has a caveat: How does one determine what constitute an adequate cumulative length of scanlines? In 1961, Hilliard & Cahn [6] assumed that intercept length is followed a Poisson distribution, and presented an equation to calculate the standard deviation of L_f, in relation to L, by

$$\sigma = \sqrt{L_f \left(\frac{\sigma'^2 / l' + l'}{L}\right)} \qquad (2)$$

where, σ is standard deviation of linear block fraction, σ' is standard deviation of intercept length, L_f is linear fraction, L is total scanline length and l' is average intercept length.

Eq. (2), has been widely used but it is valid only if L_f is less than about 5%. Medley constructed physical models with different volumetric fractions to investigate the effects of total length of scanline and L_f on the uncertainty of L_f estimation. Based on the experimental results he developed a handy chart for determining the uncertainty factor for varied scanline sampling lengths and L_f [7]. We, on the other hand, took a different track in approaching the problem while studying homogeneous and isotropic Bimrocks[8]. In the process, we have introduced the concept of "representative volume element" (RVE) and derive an analytical solution as follows,

$$\sigma = \sqrt{\frac{D}{L}}\sqrt{\frac{8}{3\pi}L_f - \frac{\sqrt{\pi L_f^{\,3}}}{2}} \qquad (3)$$

where D is an equivalent diameter of a block.

This work was further extended in this study to address anisotropic bimrocks. It is nontrivial to extend the RVE concept to anisotropic Bimrocks. In the following the basic considerations will be presented, and the derivation developed. This is followed by numerical verification.

Anisotropy in this study is represented by two factors: block shape and block orientation. Ellipsoids are mathematically simple yet sufficiently versatile for describing the shape of blocks in a bimrock. By just varying the ratios between the long, intermediate and short axes, one can model blocks with equidimentional, prismatic or platy shapes. The use of ellipsoid also allows us to define the orientation in terms of the orientations of the axes. When dealing with a two dimensional projection, an ellipsoid becomes an ellipse-which is also easy to manipulate. Our present study focuses only on using scanlines to estimate L_f in a two dimensional setting.

2. RVE CONSTRUCT

The development of the RVE concept can be described with the aid of Fig. 1. The figure shows a scanline passes through a bimrock populated with randomly distributed but similarly oriented blocks represented by ellipses. Our abstraction started by visualizing each block to be enclosed inside a represented volume. This lead us to construct an ellipse-parallelogram RVE as illustrated in Fig. 2.

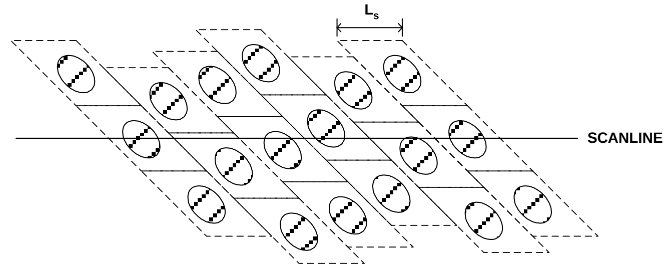

Fig. 1. Repeated ellipse-parallelogram RVE structure employed in the study.

Representative volume can be any shape as long as it is capable of tessellating a space. The parallelogram adopted here facilitated the mathematical manipulation of an arbitrarily oriented ellipse. For a given RVE, the area ratio of the block to the volume is taken to equal to the expected L_f of the bimrock. To formulate the intercepted block length in a probabilistic manner, the problem can be reformulated as one of sampling within a given represented volume. The maximum L_f achievable in an RVE so constructed is π/4. This maximum L_f corresponds to an ellipse tangent to all four sides of the parallelogram. Thus our formulation applies only for VBP no greater than π/4.

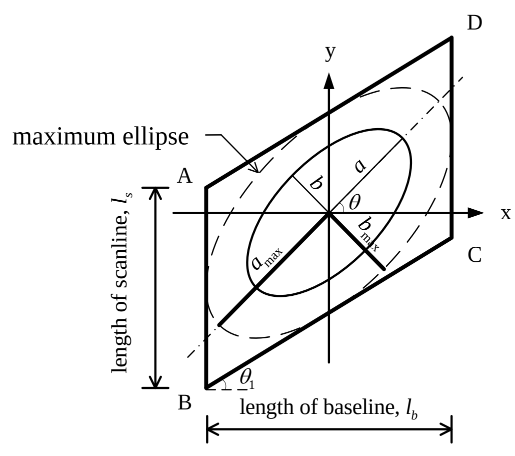

Fig. 2. Geometry of ellipse-parallelogram RVE

When the enclosed ellipse is rotated, its shape and size are kept unchanged. The parallelogram RVE used had its vertical side oriented in parallel to the scanline direction

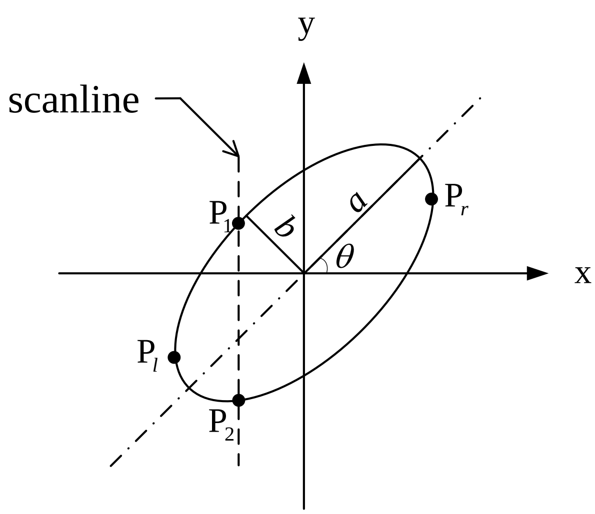

Fig. 3. Rotation of ellipse block

3. UNCERTAINTY OF INTERCEPT LENGTH IN AN RVE

3.1. Blocks are of one size

When blocks are of one size, a single RVE can be employed. By randomly passing a scanline though an RVE, the standard deviation of the intercepted length can be found as[9][10],

$$\sigma = \frac{1}{K}\sqrt{\frac{D}{L}}\sqrt{\frac{8}{3\pi}L_f - \frac{\sqrt{\pi L_f^{\,3}}}{2}} \qquad (4)$$

$$K = \sqrt[4]{k\cos^2\theta + \frac{1}{k}\sin^2\theta} \qquad (5)$$

where D is an equivalent diameter of a circle, or $2\sqrt{ab}$; k is aspect ratio (a/b) ; θ is orientation of block. K is anisotropic factor.

When K=1, Eq. (4) reduces to Eq (3) as expected.

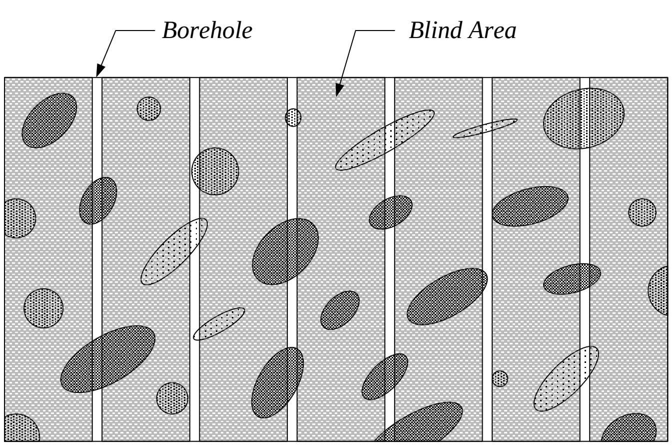

Fig. 4. Blind area in linear measurement

3.2. Blocks are of multiple sizes

To consider multiple block sizes, we introduced an equivalent diameter , D_e , defined as flows,

$$D_e = \sum_{i=1}^{n} C_i D_i \qquad (6)$$

where, C_i and D_i are proportion and diameter of the i^{th} blocks respectively. Fig 5 further illustrates the definition of the equation.

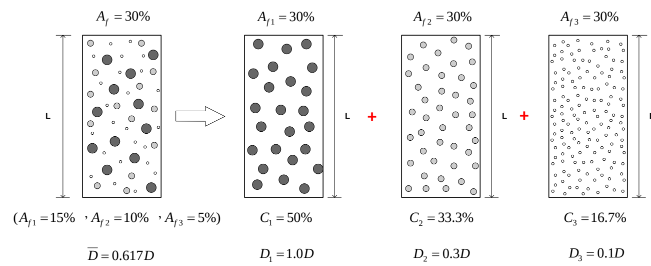

Fig. 5. Equivalent diameter can be calculated based on the diameters and proportion of blocks.

We further relate D to L and L_f, and after a length manipulation, we found that D can be removed from the equation and obtained a standard error equation below,

$$\sigma = \sqrt{L_f\left(\frac{l_e}{L}\right)}\sqrt{1 - \frac{3\pi^{1.5}}{16}\sqrt{L_f}} = \sqrt{L_f\left(\frac{\sigma'^2/l' + l'}{L}\right)}\sqrt{1 - \frac{3\pi^{1.5}}{16}\sqrt{L_f}} \qquad (7)$$

$$l_e = \frac{\sum_{j=1}^{m} l_j^{\,2}}{\sum_{j=1}^{m} l_j} = \frac{\sigma'^2}{l'} + l' \qquad (8)$$

where l_e is equivalent intercept length and l_j is the j^{th} intercept length.

4. PRELIMINARY VERIFICATION

Verification of our analytical solution is still in progress, so far our tests confirmed the validity of the solution. Several cases of numerical verifications are presented herein, followed by a field test case on gravel soils.

Firstly, we generated circular blocks with diameters ranging from 1.43 cm to 28.6 cm to simulate an isotropic bimrock as depicted in Fig 6. The verification case 1 ran scanline on this sample vertically, while verification case 2 scanline ran horizontally.

Secondly, we stretched the above sample horizontally to create a horizontally oriented

Table 1 Details of the verification cases

Case No.	Equivalent intercept length, l_e (m)	Linear fraction, L_f (%)	V (Eq. (9))		
			Numerical simulation	Eq. (7) $\times\sqrt{L}$	Hilliard & Cahn [6] Eq. (2) $\times\sqrt{L}$
1	0.121	0.357	0.121	0.128	0.208
2	0.122	0.358	0.123	0.128	0.209
3	0.122	0.365	0.123	0.128	0.211
4	0.241	0.357	0.170	0.180	0.207
5	0.0567	0.338	0.0747	0.0802	0.128

elliptical blocks as depicted in Fig 7. The verification case 3 scanned this sample with vertical scanlines, while in verification case 4, horizontal scanlines were applied. The verification case 5 scanned a gravel soil from the lateritic gravel formation, Chungli, Taiwan, as shown in Fig 8.
Results of the study are summarized in Table 1. The standard error can be written in a form that allows for a general comparison as,

$$\sigma = V/\sqrt{L} \qquad (9)$$

The factor V thus was used as a basis to normalized the standard error for comparison in Table 1. Eq. (9) of this study gave results very close to the numerical simulation, We also applied Eq. (2) from Hilliard & Cahn [6]. The ongoing verification study will investigate bimrocks over a larger ranges of V_f in extensive numerical simulation, and compare with results from physical tests.

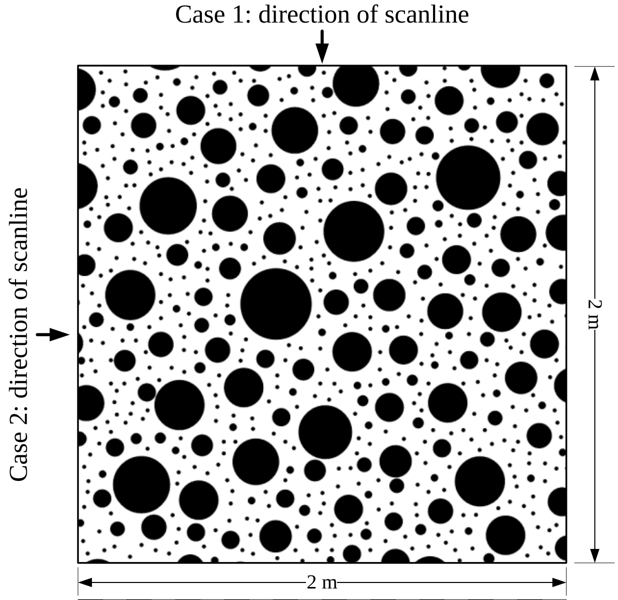

Fig. 6 Macroscopically isotropic bimrocks with circular blocks for case 1 and case 2

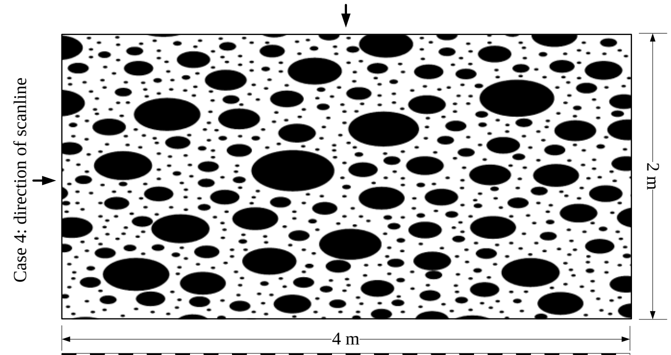

Fig. 7 Macroscopically anisotropic bimrocks with ellipse blocks(aspect ratio k=2) for case 3 and case 4

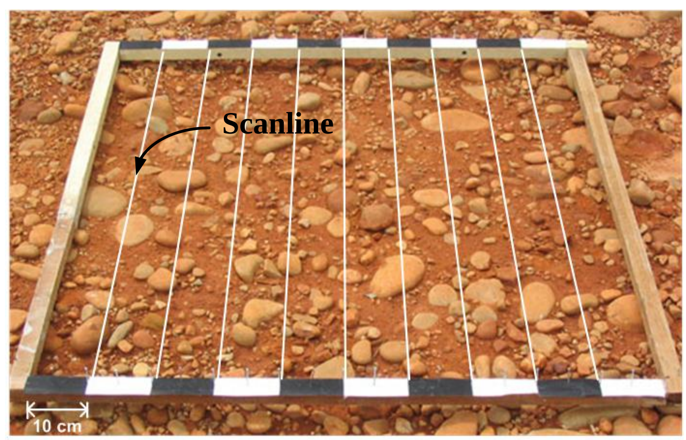

Fig. 8 Case 5 Linear measurement for Lateritic Gravel formation

5. CONCLUSIONS

We have proposed the concept of ellipse-parallelogram representative volume element and derived an analytical solution in quantifying the uncertainty of scanline estimates of volume fraction of anisotropic bimrocks. This solution includes accounts for aspect ratio, orientation of blocks, and volume fraction.

Even though our verification study is ongoing, through numerical simulations we have so far verified our results. In the process of this verification work, we have also found

- For an isotropic bimrock, orientation of the scanlines used have no impact on the mean and standard deviation of V_f estimated.
- For a given anisotropic sample, the orientation of a scanlin adopted has no impact on the expected value of V_f, but has a significant impact its uncertainty. In terms of an elliptical block, this uncertainty is smallest when the scanlines run perpendicular to the long axis, but largest when scanlines run parallel to the long axis.
- The solution by Hilliard & Cahn only applies to small volumetric fraction problems because of their adoption of Poisson distribution assumption on the intercepted length. In contrast, our formulation only limits the application of our solution to volumetric fraction no more than $\pi/4$.

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