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Uncertainty in Estimation of Volumetric Block Proportion of Bimrocks by Using Scanline Method

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ABSTRACT: Volumetric block proportion (VBP) is a crucial parameter for assessing the mechanical properties of Bimrocks. The widely used scanline method has been very successful in providing VBP estimates, but the uncertainty for such estimates has only been quantified empirically. This paper presents both analytical and numerical approaches in addressing this issue. Analytical equations were derived for the mean and variance of scanline VBP estimates based upon a representative volume element concept. Extensive numerical simulation was carried that affirmed the analytical solutions. To facilitate a general application, a normalized coefficient of variation equation is given that can be used for assessing the uncertainty of VBP. An illustrated example on its use is given at the end.

1. INTRODUCTION

Block-in-matrix rocks, Bimrocks, are defined as "mixture of rocks, composed of geotechnically significant blocks within a bonded matrix of finer texture" [1]. By this definition Bimrocks encompass a wide range of geologic materials including mélange, faulted rocks, landslide debris, and glacial till.

The overall mechanical behaviors of Bimrocks are highly dependent on their volumetric block proportion (VBP) [2][3][4][5]. An accurate estimation of VBP is, therefore, of crucial importance when dealing with Bimrocks.

Three categories of measurement methods have been used in estimating the VBP of Bimrocks, namely, one-dimensional (scanline and borehole), two-dimensional (image analyses and window mapping) and three-dimensional (sieve analyses). Although the sieve analysis is the most accurate method for laboratory-scale studies, separation of blocks from the weaker matrix is not always possible, affecting by factors such as the number and size of blocks, and the degree of contact strength between blocks and matrix [6]. Accordingly, most applications employ either one-dimensional or two-dimensional methods, or both[6]. However, the feasibility of two-dimensional methods is contingent uopn sufficient color contrast between constituent blocks and matrix. In many cases, the surface of blocks is dyed by matrix. That makes it difficult to recognize the blocks from the matrix via image analysis methods. The one-dimensional scanline method is perhaps the easiest and most efficient way for estimating VBP of Bimrocks in both laboratory and field scales.

Medley [1], Medley & Goodman [7] presented a technique of hand-tracing on a photograph of mélange cores to evaluate the convergence of VBP with respect to scanline sampling lengths. Medley [8] further prepared a series of Bimrock samples in investigating the uncertainty associated with VBP estimate. The samples were composed of Plaster of Paris matrices with known VBPs varying between 13% and 55%. Based on the comparison of measurements with known VBP values, Medley developed a chart for determining the *uncertainty factor* for varied scanline sampling lengths and VBPs[9][10].

The experiment-based works of Medley and Goodman have shown that the accuracy of Bimrock VBP estimates is influenced by many factors, among them are sampling length, block size distribution, shape and orientation of blocks as well as the VBP itself [1][7][8]. Because of the nature of their studies, these important results were empirical. As a result, it is difficult to judge to what extent their results apply. To address this important issue, a different approach is proposed. At the core of our approach is the representative volume element (RVE) concept. This makes a direct theoretical derivation possible. This is further complemented with numerical simulation.

2. METHODOLOGIES

2.1. *The Concept of Representative Volume Element*

Consider a simple case that circular blocks of a fixed size are uniformly distributed within a matrix. Given a VBP, a RVE can be thought of as a volume of square matrix contains one of the circular blocks in the middle and that the area ratio of the circle to the square equals to VBP.

A scanline of length L drawn through such a Bimrock makes some hits and some misses of the block. If the RVE is superimposed on each of the block that intersects with the scaline, as depicted in Figure 1, it becomes clear that the estimates of VBP using scanlines can be obtained by just studying one RVE. When blocks are uniformly distributed within a matrix, a scanline of length L becomes n line segments randomly drawn inside a RVE, where nL_i equals L and L_i is the width of the RVE. From this, a theoretical derivation becomes possible to estimate VBP using random variables.

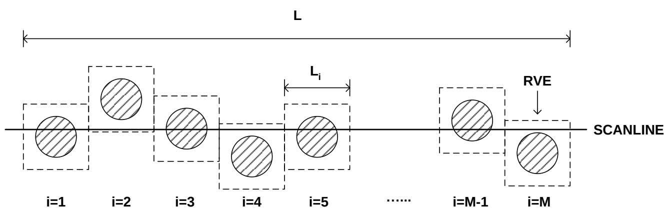


Fig. 1. Repeated RVE array structure employed in the study.

2.2. *Sample Preparation for Numerical Simulation*

We have also carried out numerical simulation to verify the theoretical results. To generate a Bimrock with circular blocks uniformly distributed with a domain of $mD \times mD$, we extend the domain in order to avoid boundary effects.

(i) Define an extended square domain of $(m+4)D \times (m+4)D$, where D is diameter of block.

(ii) Place blocks randomly within the extended domain.

(iii) Trim back to the desired square domain.

Four strips (2*D* in width) are trimmed off from 4 edges of extended square to reduce the effect of edge. This paper reports results using *m* of 30, 60 and 100, respectively. An example of square domain sample is depicted in Fig. 2.

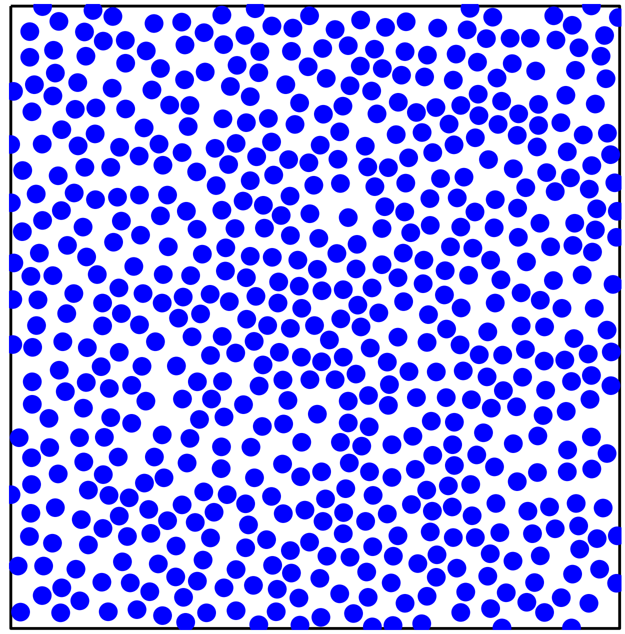


Figure. 2. An example of square domain created by random generation (*m*=30, *VBP*=49.92%)

2.3. *Analysis*

(i) Theoretical derivation based upon RVE

In the lenses of a RVE, a scanline becomes a number of line segments that randomly crosses a square. The cumulated block length intersected obtained divided by the scanline length gives the estimated VBP. As laid out, we used a square volume with a L_i by L_i dimension, and a circular block with a diameter of D. The implied VBP of a RVE in this case is $\pi D^2 / 4L_i^2$.

When a vertical line segment is drawn anywhere inside a RVE, the length of the intersection it makes in crossing a block is random variable. Let this random variable be L_c and its probability density function was found as

$$f_{L_c}(l_c) = \frac{1}{L_i} \frac{l_c}{\sqrt{D^2 - l_c^2}} \tag{1}$$

It can further be shown that the mean and variance of Lc are

$$E[L_c] = \frac{\pi D^2}{4L_i} \tag{2}$$

$$Var[L_c] = \frac{2}{3}\frac{D^3}{L_i} - \frac{\pi^2}{16}\frac{D^4}{L_i^2} \tag{3}$$

Since a scanline is a repetition of many segments of L_i in length, a division of L_c with L_i thus gives the basis for estimating mean and variance of the VBP estimated:

$$E[VBP] = \frac{\pi D^2}{4{L_i}^2} \qquad (4)$$

$$Var[VBP] = \frac{2}{3}\frac{D^3}{{L_i}^3} - \frac{\pi^2}{16}\frac{D^4}{{L_i}^4} = \frac{16}{3\pi^{1.5}} E[VBP]^{1.5} - E[VBP]^2 \qquad (5)$$

The probability distribution of VBP can also be derived form as Eq. (1) as follows,

$$f_{VBP}(v) = \frac{v}{\sqrt{\frac{4}{\pi} E[VBP] - v^2}} \qquad (6)$$

In the present layout, RVE has the VBP has an upper bound VBP of π /4.

(ii) Statistics of a scanline VBP estimates

Each scanline can be viewed as a summation process, thus the statistics of its VBP will converge the Gaussian distribution that can be explained by the *Central Limit Theorem*. Let each of the scanline used be of the same length of $L=nL_i$, its estimated VBP, denoted with a subscript G as, VBP_G, can be obtained by summation over n independent identically distributed random variable as follows

$$VBP_G = \frac{1}{nL_i}\sum_1^n L_c \qquad (7)$$

The mean and variance of VBP therefore becomes, $E[VBP_G] = E[VBP]$

$$Var[VBP_G] = \frac{1}{n^2} Var[VBP] \qquad (8)$$

and the standard deviation becomes,

$$\sigma[VBP] = \frac{1}{\sqrt{L/D}}\sqrt{\frac{8}{3\pi}E[VBP] - \frac{\sqrt{\pi\ E[VBP]^3}}{2}} \qquad (9)$$

This can be rewritten in terms of a normalized coefficient of variation, COV, as

$$COV[VBP]\sqrt{L/D} = \sqrt{\frac{8}{3\pi\ E[VBP]}} - \sqrt{\frac{\pi}{4E[VBP]}} \qquad (10)$$

(iii) Numerical Simulation

Numerical simulation has been carried out using single scanline as well as group of scanlines. The results are discussed in the following section.

3. RESULTS

3.1. VBP Distribution estimated using scanlines

As an illustration, we have employed scanlines of different lengths in the simulation. Herein the results with mean VBPs of 5% and 50%, respectively, are reported. Different lengths of scalines were generated with L/D ratios varying between 30 and 1000.

Fig. 3 shows the histograms of estimated VBP obtained from RVE, converge to a Gaussian distribution as L/D increases. From this, in the field, we can use the estimated VBP and the average block size to get a picture of the uncertainty involved. Given an estimated VBP, Eq. (4) can be used in estimating L_i with an average block size value for D observed in the field. Eq (4) can then be used to estimate the size of RVE, L_i. If a scanline is selected for 30 times of L_i, one obtain an equation as a function of VBP :

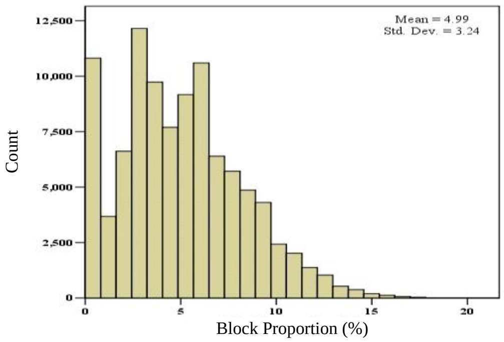
$$L = 30L_i = 26.6 \times \sqrt{1/VBP}\, D \qquad (11)$$

If *VBP*=5%, the minimum, this length would be 119*D;* whereas for *VBP*=50%, it corresponds to 38*D*. One reason for this difference is that at small VBP value, unless a scanline is long enough, many will turn up without hitting the block. Thus a higher VBP converges to a Gaussian distribution faster. In general application, however, Eq (11) would be often sufficient.

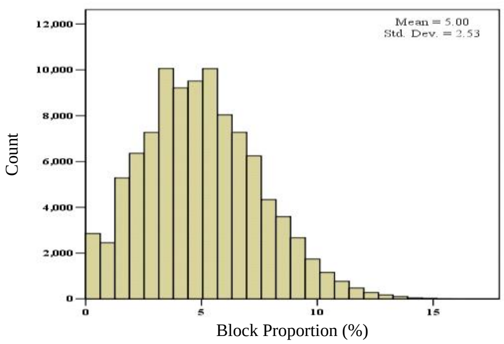
3.2. Mean, Standard Error

Standard error of VBP estimates of Bimrocks are affected by the length of a scanline used, and by the VBP. Eq. (10) provides a theoretical basis for addressing this issue. To shed further insight of the implication of Eq. (10), we have conducted extensive simulations using samples of different VBP, and employing scanlines of different lengths. The results are summarized in Fig. 5. These data match well with Eq. (9) predictions which are shown as curves.

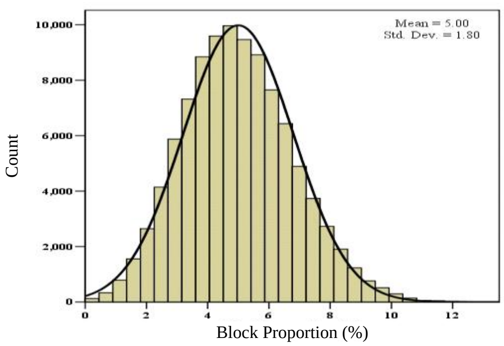
The simulation data of Fig. 5, when normalized, are also in agreement with the theoretically derived Eq. (10). Eq. (10) is further plotted in Fig. 6 to facilitate applications.



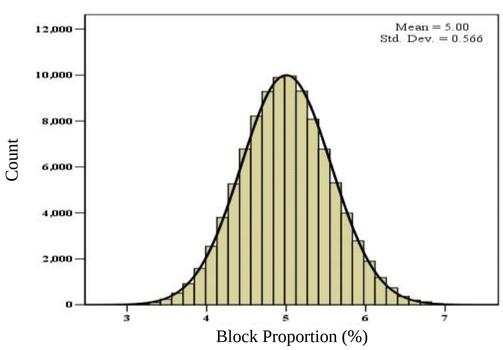
(a) L/D=30



(b) L/D=50

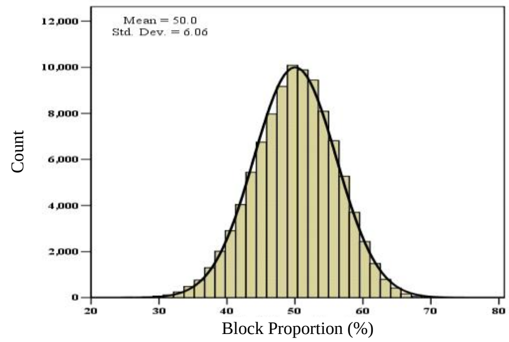


(c) L/D=100

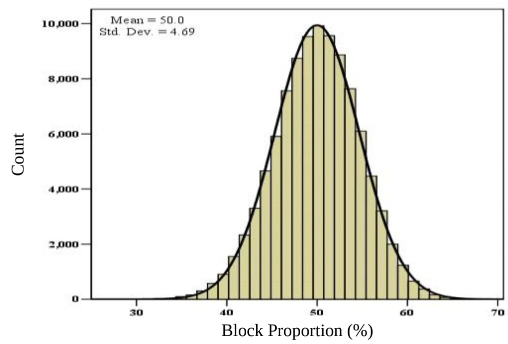


(d) L/D=1000

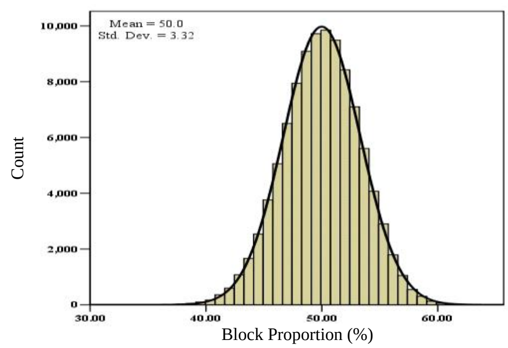
Figure 3. Histograms of VBP estimates with different lengths of scanlines with mean VBP of 5%.



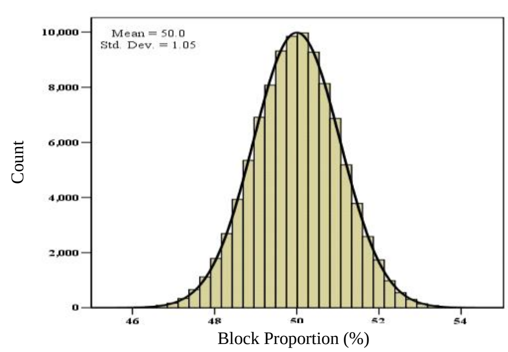
(a) L/D=30



(b) L/D=50



(c) L/D=100



(d) L/D=1000

Figure 4. Histograms of VBP estimates with different lengths of scanlines with mean VBP of 50%.

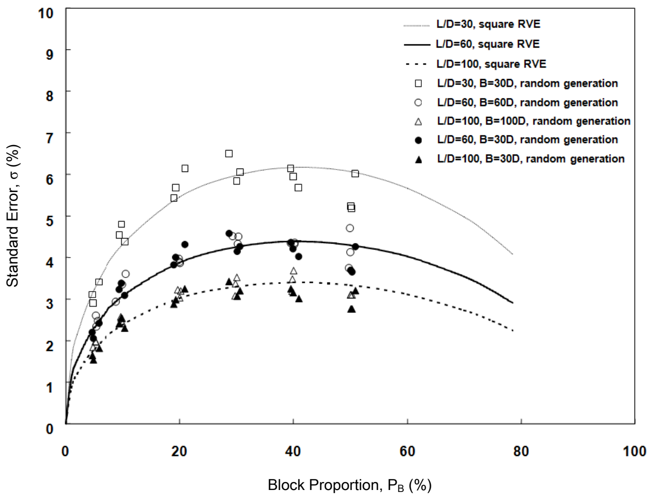


Fig. 5. Standard errors obtained through numerical simulation

$$COV[VBP]\sqrt{L/D} = \sqrt{\frac{8}{3\pi\ E[VBP]} - \sqrt{\frac{\pi}{4E[VBP]}}}$$

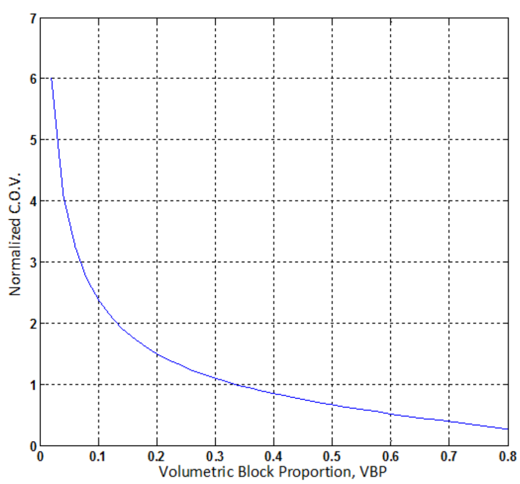


Figure 6 VBP versus normalized C.O.V. plot.

4. AN ILLUSTRATED EXAMPLE

Even though this study was mainly constructed on the basis of a simple RVE, sensitivity study involving other types of RVE have been carried out and the applicability of the current results affirmed. Here we examined a published example of VBP estimate from Scott Dam, North California [9]. The largest block encounter in the Scott Dam was estimated to be about 30 m long. About 360 m of exploratory drilling had been performed during the life of the dam, but only about 150 m of core had been recovered. The block proportion was estimated to be about 40% [9]. Conservatively, we used here the largest block size as the estimate for D, and the effective sampling length *L/D*, thus was only 5 (i.e.,150/30). Using Eq. (10) or Fig 6, the COV was found to be 0.38. Taking a 68% confidence level would give the VBP estimates to be within the range of (1±0.38)×40%, or between 24.8% and 55.2%. Since L/D of 5 was rather small, the error was bound to be significant. However the actual L/D ratio might be higher and accordingly error smaller, nonetheless, without knowing more details about the survey, a conservative estimate might be warranted.

But if one would assume a longer drilling length, for instance, the core taken was of 600 m long. With this, *L/D*= 20, then the COV would have been 0.19. Taking the same confidence level, the estimates would instead lie within a much narrower range of 32.4% to 47.6%.

Eq.(10) thus can also serve as a basis for cost-benefit analysis in determining the length of a scanline required in order to reach a certain estimate precision.

5. CONCLUSIONS

This study presented an innovative way to quantify the uncertainty of volumetric block proportion estimates of Bimrocks using scanlines. The present theoretical derivation was validated by numerical simulations. Moreover, a single normalized coefficient of variation equation is found to be applicable to scanlines of various lengths. The present results have important practical implications. This can readily be observed from an example is provided that illustrates the application and benefits of the proposed method.

Follow-up investigations on the effects of block orientation and block shape impacts have shown that the current results are still valid. A study on the impact of block size distribution is still on-going.

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